Traffic circles

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Abstract

The use of a traffic circle is a relatively common means of controlling traffic in an intersection. Smaller Traffic circles can be especially effective in routing and controlling lower levels of vehicular flow, since traffic flows in only one direction within the circle. In larger intersections, however, the situation becomes more complicated and congestion can easily form at higher traffic levels. In order to find a means of efficiently directing traffic within a traffic circle, we have written a continuous car-following traffic circle simulation program based on the traffic dynamics described by Bando et al. By dividing the traffic circle into different sections, we are able to apply SAGE's digraph analysis tools to find the most efficient traversal paths at specific speeds. This allows us to realistically model car interactions within the traffic circle in order to determine the most efficient methods for controlling the traffic circle. After analyzing the results from our analysis, we determined that the best control methods for small traffic circles is for cars already within the traffic circle to yield to cars entering the traffic circle. This is significantly more efficient than either having either no control or having incoming vehicles yield to cars already in the traffic circle. In the single lane traffic circle, we find that the minimum average travel time is approximately 11.26 seconds for cars entering at a rate of $\frac{1}{3}$ veh/s into a single lane traffic circle of $40m$ radius.

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1 Introduction

Traffic circles have been used to regulate both vehicular traffic and non vehicular traffic in developing cities around the world. Their effectiveness has been observed in both their efficiency and their ability to manage relatively large volumes of traffic and reduce accidents because all vehicles drive in the same direction.

1.1 Problems with Existing Traffic Circles.

Traffic circles around the world can vary significantly in design, though most feature traffic entering from intersecting streets and flowing in a single direction around a center island. Traffic in smaller traffic circles is usually fairly efficient since traffic density is usually minimal and congestion is avoided since cars need not, and often are not allowed to, switch lanes often. In larger traffic circles however, traffic becomes considerably more congested due the the high traffic volume and the necessity to switch lanes.

1.2 The Goal of Our Model

The goal of this model is analyze the effectiveness of different traffic control devices on traffic circles purely in terms of efficiency of cars moving through them. Since the behavior of traffic within a traffic circle varies considerably with the parameters of the traffic circle, we have conducted simulations on an expansive set of different traffic circle designs in order to learn the most efficient control for each.

First we need to develop definitions of the quantities we wish to optimize. The following definitions are adapted from [1].

Definition 1. Flow rate (in veh/hr) is how many vehicles go through a specific area in a unit of time.

Definition 2. We term the **Average Travel Time** (ATT) of a traffic system to be the average time it takes a vehicle to exit after having entered the system.

Thus the goal of this model is to determine what use of traffic devices at a particular traffic circle have the lowest ATT and highest capacity at a given entrance flow.

2 A Theoretical Model for Traffic Flow

Most traffic flow models concern themselves with either the microscopic scale or the macroscopic scale. Macroscopic models attempt to model traffic by comparing it to fluid dynamics or particle dynamics. Microscopic models instead deal with individual cars and drivers, and usually fall into continuous or cellular automata models. For traffic situations with sensitive dependence on system disturbances though, continuous models have been found to be more useful and accurate [3]. Due to the unstable nature of traffic within a traffic circle, we decided to use a continuous model based on the following Bando traffic model.

2.1 Car-Following Traffic Flow with Bando Acceleration

A particularly well known car-following model for traffic patterns was described by Bando et al in [2]. This model is well known to show complicated congestion patterns and is relatively simple with only several parameters [2][3], for which appropriate values have been established. We define x_i to be the position of car i. Then the model predicts that a vehicle will alter its acceleration to so that an ideal headway, which depends on the velocity of the car and the position of the car ahead of it, is maintained. The differential equation governing the linear motion is then

$$x_i''(t) = k[V(\Delta x_n(t)) - x'(t)],$$

where Δx_n is the headway between cars i and $(i-1)$ and $V(\Delta x_n)$ is the optimal velocity function and is defined as

$$V(\Delta x_n(t)) = V_1 + V_2 \tanh[C_1(\Delta x_n(t) - l_c) - C_2].$$

Helbing et al, estimated the values of these parameters as $V_1 = 6.75\text{m/s}, V_2 = 7.91\ \text{m/s}, C_1 = .13\text{m}^{-1}$ and $C_2 = 1.57$.

For a system with a considerable volume of vehicle traffic, the nonlinear nature of this system combined with the large number of variables makes it largely analytically intractable, though we can find the stability of the steady state solutions.

2.2 Stability of the Bando Model

It is important to ask whether the Bando model is stable, i.e. whether the solutions for equally spaced cars traveling on a circular track, all with the same velocity, and such that $x'(t) = V(\Delta x)$ for each car, is a stable solution. If one solves the linearized system, one finds that the steady state solution to be stable if $V'(b) < \frac{k}{2}$ and unstable if $V'(b) > \frac{k}{2}$ where b is the steady state headway distance [2]. If we evaluate our expression for $V'(b)$ using the stated parameter values, we find that this steady state solution to the model is stable in a circle of radius 40m (the size of the traffic circles we tested) when there are 10 or fewer cars in the circle.

3 Modeling Traffic Inside a Traffic Circle

We wrote out model in Python and used SAGE for mathematical simulation. In this section we outline our model.

3.1 Basic Assumptions for our Model

There were several assumptions that we needed to make in order to construct the model:

- Every car has a destination before it enters the traffic circle, and is always trying to reach its exit
- Traffic should flow in only one direction around the traffic circle.
- The traffic circle should be divided into lanes.

3.2 Traffic Circles

Due to limitations in computation time, we designed two traffic circles on which to test varying types of traffic control devices. Pictures and descriptions are shown below.

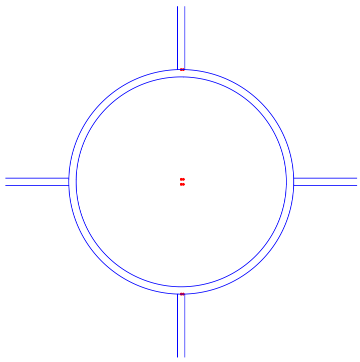

Figure 1: A rendering of the simplest single lane traffic circle.

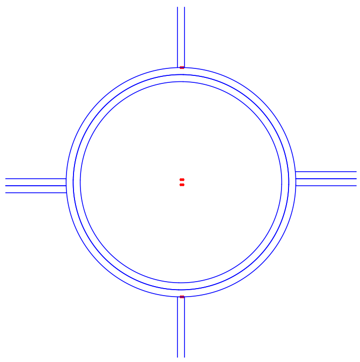

Figure 2: Two lane traffic circle with two, single-lane streets and two, two-lane streets.

3.3 Traffic Flow

For most situations within the traffic circle, we numerically applied the Bando traffic dynamics described in the previous section. In programming the model, we had to alter the dynamics somewhat so that the headway for a car was not always calculated based on the car in front of it, but sometimes on cars in other lanes or on traffic control devices.

3.4 Traffic Control Devices

The traffic control devices that are can be present in a traffic circle are

- traffic lights,
- stop signs,
- yield signs,
- and pedestrian signals.

These traffic control devices can essentially be placed anywhere on the circle, and in our simulation, we tested which combinations were most efficient for different traffic circle situations.

3.5 Switching Lanes and Digraphs

Though drivers are usually advised against switching lanes within an traffic circle, such behavior becomes unavoidable in very large circles. In traffic circles, there are many situations where cars have to switch lanes. For example, cars may need to switch in or out of an exit lane, or other lanes may be faster. In our model, we divided the traffic circle into separate segments of the road. A car can drive within a segment using the rules described above. When a car needs to move to a different segment of the road, a *map* is used to move it. The different segments of the road and the maps that connect them constitute the nodes and paths of a *digraph*. A digraph, or directed graph, that can only be traversed by cars in the direction of the arrows. In our model, each exiting street lane, entering street lane, and each quarter lane around the circle were treated as nodes. The process of deciding and executing a lane change is outlined below.

- Find all traversals of digraph to desired exit.
- Evaluate which traversal is fastest at current speed.
- Wait until opening in other lane is large enough for car.
- Make lane change.

The actual lane change takes a certain amount of time, during which the car is essentially counted as being in both lanes in order to compute the dynamics.

Below is the simplest one lane traffic circle with single lane intersecting roads shown below alongside the corresponding digraph.

In order to for cars to decide when to switch lanes, we had SAGE determine which path would be fastest given the current velocities of cars within each segment of the traffic circle. Note that although the traffic lanes are divided into segments, our model still simulates the position of cars within each segment continuously.

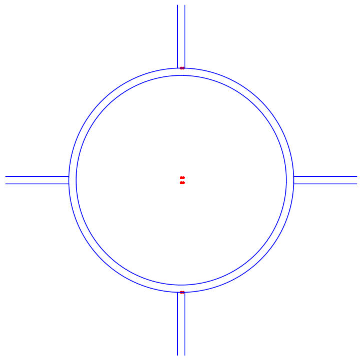

Figure 3: A rendering of the simplest single lane traffic circle.

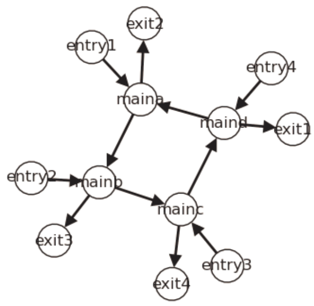

Figure 4: The corresponding digraph for the single lane traffic circle.

3.6 Entering and Exiting the Traffic Circle

In our model, streets entering the circle are modeled similar to the lanes within a traffic circle. Entering the circle is modeled by a map into one of the lanes and is treated similar to merging.

3.7 Pedestrians

Pedestrians are assumed to want to cross to the center island and across the entrances and exits to the center island. Though pedestrians are not explicitly modeled in the dynamics, it is assumed that pedestrian signals are set up at any lights within the circle and along any of the entrances or exits. We make no calculations for pedestrians illegally crossing the traffic circle.

4 Applying the Model

In testing these traffic circles with varying traffic control devices, we able to get results for average travel time (ATT) and exiting flow through the traffic circle and how they were dependent on entering flow and the specific traffic control devices. The results are summarized on a case by case basis.

4.1 Single Lane Circle with Single Lane Streets

The traffic circle, as displayed in Figure 1, was tested with no control, yield signs for entering traffic, and yield signs for traffic already in the circle. All tests were run with a constant flow of entering cars at $\frac{1}{3}$ cars/sec at each entrance. The results are shown in the following figure.

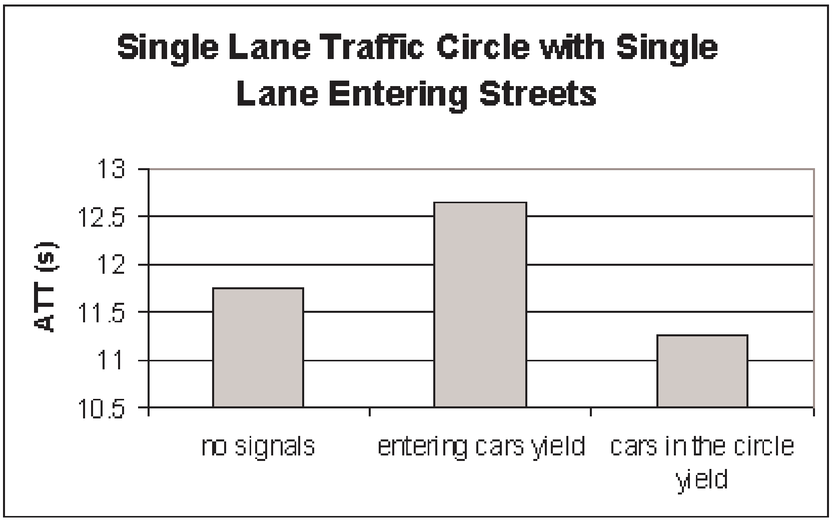

Figure 5: Average travel time for the single lane traffic circle.

As one can see, the lowest average travel time for cars is achieved when cars already within the traffic circle yield to cars entering the traffic circle. This is interesting because conventionally cars entering the traffic circle yield to cars already within the traffic circle.

5 Traffic Summary

The effectiveness of traffic circle control mechanisms can be effectively measured using computer simulations. A traffic engineer may use our discrete traffic flow model to best calculate which control mechanism suit any given traffic scenario. In the most basic case, a traffic circle consisting of one lane with four single entry lanes symmetrically placed, entering the circle, we find that cars yielding to traffic already in the circle produce the lowest average time of traveling through the traffic circle. Our findings are limited by the assumptions of our model. The findings assumed cars travel in circular lanes in low traffic density with relatively few lane changes. In consequence, the results may not hold for congested traffic.

References

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[2] Bando, M. and Hasebe, K. and Nakayama, A. and Shibata, A. and Sugiyama, Y. ”Dynamical model of traffic congestion and numerical simulation”. *Phys. Rev. E.* 51 (2): 1035-1042. 1995.

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